Research Summary

TEACHING FRACTIONS IN MIDDLE GRADES MATHEMATICS

In support of *This We Believe* characteristic:

- Educators who value working with this age group and are prepared to do so
- Multiple learning and teaching approaches that respond to student diversity
- Curriculum that is relevant, challenging, integrative, and exploratory

Fractional concepts are important building blocks of elementary and middle school mathematics curriculum. Conceptually based instruction of fractions requires teachers to have a complete understanding of the subject matter. Several researchers (e.g., Ball, 1990; Shulman, 1986; Wilson, Shulman, & Richert, 1987) have proposed theories about teacher knowledge characteristics and structure. In the area of mathematics, Hill, Schilling, and Ball (2004) have extended Shulman's original ideas about pedagogical content knowledge and have developed a model for mathematics' teacher knowledge referred to as mathematics knowledge for teaching (MKT). In their model, the three knowledge domains most central to mathematics teaching are *common* knowledge of mathematics, *specialized* knowledge of content, and knowledge of students and their ways of thinking about the content. Hill and associate's model provides a theoretical base for this research summary of what middle school teachers need to know to teach fractions. *Common* knowledge is the knowledge that a mathematically educated adult, not necessarily a teacher, needs to possess to provide correct mathematical solutions. *Specialized* knowledge of content is the possession of mathematical knowledge and skills such as being able to explain why an algorithm works or being able to provide students with multiple representations addressing diverse learning styles. Middle school mathematics teachers need both common and specialized knowledge of fractional concepts along with general pedagogical knowledge.

The development of students' knowledge of fractions starts in elementary school and progresses to middle school. This progression of fractional concepts into the middle school curriculum requires students to transition from additive to multiplicative reasoning, develop proportional understanding, and understand the relationship between fractions, ratios, and proportions (Sowder et al., 1998). To help students transition from basic understanding of beginning fractional topics to more advanced concepts that are centered on proportional reasoning, middle school mathematics teachers need to understand how students develop multiplicative and proportional reasoning (Sowder et al.). These teachers also need in-depth understanding of fractional concepts to provide students with multiple representations and experiences that enhance reasoning about fractional relationships and operations (Ball, 1993; Siebert, 2002; Taber, 2002).

Multiplicative and Proportional Reasoning

An important developmental step for middle school students is the transition from additive to multiplicative reasoning (Kent, Arnosky, & McMonagle, 2002; Sowder et al., 1998; Vanhille & Baroody, 2002). To scaffold this transition, teachers need to understand the differences between additive and multiplicative reasoning (Sowder et al.). In accordance with Shulman's (1985) belief that "to be a teacher requires an extensive and highly organized body of knowledge" (p. 47), middle school teachers need to have an in-depth understanding of multiplicative reasoning so that they can explain complex ideas of multiplicative concepts conceptually, adapt their instruction for diverse learning styles and ability levels, and provide students with contexts to reason multiplicatively without relying on additive approaches.

The type of comparison students make to explain changes in quantities characterizes the differences between additive and multiplicative reasoning. When comparing additively, the focus is on the difference between the quantities; whereas, in multiplicative comparisons, the focus is on the rate of change. With additive reasoning students interpret the change in quantities such as how many more, which results in finding the difference, even when multiplicative comparison is required (Harel & Behr, 1990). Students who use multiplicative reasoning, on the other hand, can differentiate between situations that require additive or multiplicative transformations (Harel & Behr). Although additive reasoning can develop intuitively, multiplicative reasoning is difficult for students to develop and often requires formal instruction (Sowder et al., 1998). Therefore, it is essential for middle school mathematics teachers to (a) know how students develop multiplicative reasoning, (b) capitalize on students' prior knowledge of multiplicative concepts such as multiplication and division of whole numbers to strengthen students' multiplicative reasoning, and (c) provide students with learning contexts and experiences that encourage

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multiplicative comparisons to prepare them for higher-level mathematics topics.

Because multiplicative reasoning fosters understanding of fractions and proportions, fraction instruction also needs to promote multiplicative reasoning (Sowder et al., 1998; Vanhille & Baroody, 2002). For example, in their discussion about additive and multiplicative reasoning of fractions, Thompson and Saldanha (2003) suggested that the former is frequently inadequate. Additive understanding of fractions involves thinking about *a/b* as "*a* parts out of *b* parts." This leads students to think that the numerator is included in the denominator, and makes it hard for them to interpret fractions such as 4/3 because 4 is not included in 3. Multiplicative understanding of fractions, on the other hand, is based on thinking about two quantities as measured in units of each other. For example, M = (a/b) x N means that "M is a/b as large as N" or "N is b/a as large as M." This reasoning does not imply any inclusion of the numerator in the denominator and provides a more adequate understanding of fractions (Thompson & Saldanha).

Teachers can also focus students' attention on multiplicative reasoning as they teach fraction equivalency. Fractions involve *between-* and *within-multiplicative relations* (Vanhille & Baroody, 2002). Within-multiplicative relation refers to the relation between the numerator and the denominator of a fraction. For example, in ¾ the numerator, 3, is ¾ of the denominator, 4 (i.e., 3 = 3/4 x 4); and the denominator, 4, is 4/3 of 3 (i.e., 4 = 4/3 x 3). All equivalent fractions share the same within-fraction multiplicative relation. Between-multiplicative relation refers to the relation between the numerators and the denominators of equivalent fractions. If two fractions are equivalent, the ratio between the numerators is the same as the ratio between the denominators.

Understanding the ratio, which is essentially the result of the comparison of two quantities with multiplicative relationships, plays an important role in the development of multiplicative and proportional reasoning (Sowder et al., 1998; Post, Cramer, Harel, Kiernen, & Lesh, 1998). In a study with inservice teachers, Sowder and associates found that teachers who solved multiplicative problems correctly used ratio and proportional reasoning and concluded that the development of ratio and proportion concepts is important for solving problems involving multiplicative relations.

Proportional Reasoning

To help students understand the meaning of ratio and proportion, teachers need to know the ways in which students develop proportional understanding and provide students with conceptual curricula and meaningful classroom experiences to become proportional reasoners (Lamon, 1999). According to the National Council of Teachers of Mathematics [NCTM] (1989) *Curriculum and Evaluation Standards,* "[Proportional reasoning] is of such great importance that it merits whatever time and effort that must be expended to assure its careful development" (p. 82). It is important that middle school mathematics teachers have the necessary content and pedagogical knowledge and skills to be able to build on students' informal knowledge of proportionality, promote proportional thinking using effective questioning, and provide multiple examples and accurate representations to convey different proportionality concepts. Cramer, Post, and Currier (1993) suggested that teachers need to engage students in hands-on experiences with ratio and proportional situations. The learning of procedural knowledge used in proportions, such as the traditional cross-multiply-and-divide algorithm, should be postponed until students conceptualize the underlying meaning of these concepts (Cramer et al.). At the beginning of proportion instruction, it is important that teachers use and build on students' intuitive understandings and thought processes (Cramer & Post, 1993). As students develop conceptual understanding, teachers should provide different strategies and algorithms.

Conceptual Understanding of Fractional Operations

Although most of the fractional operations—addition, subtraction, and multiplication—are covered in elementary school, they are often revisited in middle school. Research suggests that students have a procedural knowledge of fractional operations rather than an understanding of underlying concepts (Mack, 1990). Therefore, middle school teachers need to possess a conceptual understanding of fractional operations to deliver a sense-making curriculum (Sowder et al., 1998). For example, to meaningfully add and subtract fractions, Mack (1995) has argued students must interpret fractions as a number—a value of a quantit—instead of thinking of fractions as two numbers (Kerslake, 1986).

NCTM (n. d.) asserted that "multiplication by fractions and decimals can be challenging for middle grades students if experiences with multiplication by whole numbers have led them to believe that multiplication makes bigger" (¶1). To communicate the effects of multiplication by numbers less than 1, teachers can use concrete models such as manipulatives. Furthermore, even if students are capable of solving problems such as John has 4 cakes and Ann has 2/3 as many cakes as John, students think of these problem situations as multiplication rather than division because the result is smaller (Taber, 2002). Therefore, Taber suggested that instruction of multiplication with fractions should relate to multiplication with whole numbers while reconceptualizing students' understanding of whole-number multiplication to include fractions as multipliers (Taber). Teachers need to have the necessary specialized knowledge, such as knowledge of a variety of representations including concrete models and real-world problems, to help students visualize or relate as they transition from multiplication by whole numbers to

multiplication by fractions (Taber). Division of fractions, which generally is taught in the middle grades, requires closer consideration.

Division of Fractions

Division of fractions is one of the concepts in upper elementary and middle school mathematics curriculum where students lack conceptual understanding. This is because division of fractions is taught primarily by emphasizing the traditional invert-and-multiply algorithm without providing justification as to why it works (Siebert, 2002). Helping students develop conceptual understanding of division of fractions requires teachers to have substantive understanding of the procedures with their underlying meanings and their connectedness to related mathematical topics (Ball, 1990). Teachers need to have insights into students' thinking processes—their conceptions and misconceptions about division of fractions—to facilitate their understanding (Tirosh, 2000).

In a study of U.S. and Chinese teachers' knowledge of dividing fractions, Ma (1999) found that U.S. teachers relied heavily relied on the traditional algorithm and lacked conceptual understanding to generate appropriate representations. Research on preservice teachers' understandings has also shown that their knowledge is often procedural and fragmented (Ball, 1990; Borko et al., 1992; Simon, 1993). In Ball's study about how preservice teachers understood division of fractions, she found that almost all of them were able to compute using the traditional invert-and-multiply algorithm; however, only a few were able to develop a mathematically appropriate representation. Borko and associates' case study supported Ball's findings and found that such limited understanding prevented one student teacher from providing a conceptual explanation for the traditional algorithm.

Although the traditional algorithm may be the most efficient way of dividing with fractions (Bergen, 1966; Capps, 1962), it is based on the inverse nature of division and multiplication, which requires algebraic reasoning. However, division of fractions typically is taught before algebra, and, thus, this approach is not readily accessible to every student (Sharp & Adams, 2002). Teachers should introduce the fraction division concept by building upon students' previous knowledge of whole number division using multiple representations. Ma (1999) suggested that connectedness is an important part of understanding. Teachers with substantive understandings of division of fractions make explicit connections to students' previous knowledge of measurement and partitive interpretations of division (Ball, 1990; Flores, 2002). Using multiple representations such as problems with real-world contexts or pictures to convey these interpretations can provide meaningful mental images (Sharp & Adams, 2002; Siebert, 2002). Because students usually learn division of whole numbers through subtraction, division with fractions can build upon the repeated-subtraction definition of division (Sharp & Adams). Another procedure for division of fractions that emerges from whole numbers is the common-denominator algorithm. Sharp and Adams found that students who were given the opportunity to construct personal knowledge of division of fractions most frequently developed the common-denominator algorithm.

After students explore the meaning of division with fractions, they are encouraged to adopt more efficient ways—such as the invert-and-multiply algorithm—to perform the process. Conceptual understanding of the invert-and-multiply algorithm requires understanding division as the inverse operation of multiplication, with integers and fractions (Ball, 1990). The reasoning should be made explicit to students by using examples such as 8 / 4 and 8 x 1/4, which result in the same answer. Teachers can also link the invert-and-multiply algorithm to measurement and partitive interpretations of division using pictorial representations to show them how inverting and multiplying actually leads to division (Siebert, 2002).

Teachers need to have a thorough and flexible understanding of division of fractions to appraise and extend students' intuitive understandings and student invented methods. This requires both subject matter knowledge and understanding of cognitive processes, whether correct or incorrect, used by students in dividing fractions. In her study on preservice teachers' knowledge of students' conceptions about division of fractions, Tirosh (2000) found that before studying students' ways of thinking, preservice teachers attributed students' common mistakes to algorithmically-based errors. However, after becoming familiar with current research findings on students' thinking, they became aware of students' intuitively-based misconceptions about division of fractions such as their tendency to attribute properties of operations with whole numbers to fractions.

Conclusions and Recommendations

Multiplicative and proportional reasoning are the fundamental ideas of many of the mathematics topics studied in grades 6–8 (NCTM, 2000). Middle school teachers need to possess a fluid and flexible understanding of these topics to present fractional concepts in an environment that links fractions to proportional reasoning. Middle school teachers should also be equipped with the necessary knowledge to help students develop conceptual understanding of fractional concepts such as accurate and appropriate representations for particular purposes, effective questioning techniques to promote multiplicative and proportional reasoning, and connected knowledge to build on students' prior knowledge. Another important aspect of teacher knowledge is middle school students' conceptions and misconceptions of fractions and the possible conceptual obstacles as they transition from

operations, specifically multiplication and division, with whole numbers to operations with fractions (NCTM, n. d.).

Research on what teachers know about fractions and knowledge of proportional reasoning is significantly less widespread than research on students' understandings of these concepts. Further research needs to be conducted in the following areas: (a) teachers' misconceptions of fractions and proportional reasoning, (b) linking teacher knowledge to the research on student understanding of fractions, and (c) effective classroom pedagogy to help students develop fluency in multiplicative and proportional reasoning.

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Carpenter, T. P., Fennema, E., & Romberg, T. A. (Eds.). (1993). *Rational numbers: An integration of research.* Hillsdale, NJ: Erlbaum.

This edited book is concerned with the construction of integrated programs of research on teaching, learning, curriculum, and assessment of rational numbers, with the goal of having a significant impact on current reform in school mathematics. The volume draws on relevant studies with a focus on identifying and integrating research paradigms in rational number concepts. Another focus of the book is to understand the effects of instruction on students' understanding of rational number concepts. The research discussed in this volume is tied together by the common content strand of rational number and goes beyond previous work that solely focused on student thinking or content analysis and made it difficult to identify curricular or instructional implications. The goal of many of the chapters—studying and understanding the relationships between teaching and learning of rational numbers—makes the book a relevant resource for mathematics educators as well as researchers.

Lamon, S. (1999). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers.* Hillsdale, NJ: Erlbaum.

This book was written to bring the current research findings on students' thinking and learning of rational numbers to life in classrooms and is a relevant resource for teachers, researchers, and curriculum directors. The book was developed with the intent of facilitating conceptual understanding of rational numbers rather than relying on traditional algorithms and procedures. The book provides instructional methods and materials that can be used with elementary and middle school students to develop proportional reasoning. The activities also facilitate teachers' understanding of students' proportional reasoning processes and their use of instructional approaches to communicate complex concepts in rational numbers. Each chapter contains sample questions, a variety of activities, and students' solution strategies that can be analyzed. The activities have been tested with students in grades 3–8 and with both preservice and inservice teachers. A companion book discusses all the solutions in depth and provides further examples (see recommended resources below).

Litwiller, B., & Bright, G. (Eds.). (2002). *Making sense of fractions, ratios, and proportions: 2002 yearbook.* Reston, VA: National Council of Teachers of Mathematics.

This 2002 NCTM Yearbook contains 26 scholarly articles that provide insights into how elementary and middle school students think about fractions, ratios, and proportions. The book is divided into three sections—Elementary School, Middle School, and Professional Development. The authors provide insights and suggestions to transition students from elementary fractional topics to middle school concepts centered on proportional reasoning. Multiplicative and proportional reasoning are stressed throughout the book, in preparation for higher-level mathematics topics. The issues covered in students' learning and understanding of fractions, ratios, and proportions make this book a relevant source for curriculum leaders in both schools and universities. Additionally, the professional development section is a valuable resource for mathematics educators. A companion booklet entitled *Classroom Challenges* contains student activities and teacher handouts that can facilitate and reinforce the application of the ideas presented by the authors in the classroom.

The Rational Number Project. (n.d.). Retrieved August 27, 2007, from http://www.education.umn.edu/rationalnumberproject/

This Web site includes an extensive bibliography generated by the faculty members associated with the Rational Number Project (RNP) for the last 20 years, making it the longest sustained cooperative, cross-university research project in mathematics education history. The RNP Web site contains more than 90 publications including articles, book chapters, and several books dealing with the teaching and learning of rational number concepts including fractions, decimals, and ratios. This site also includes research on proportionality, with explicit consideration to proportional reasoning. The collection of literature within this Web site is useful for researchers interested in research on rational number teaching and learning or multiplicative structures including proportional reasoning. The Web site also provides middle grades fraction lesson plans, making it a practical resource for mathematics educators. The RNP research findings can be used both to improve teaching practices and to identify possible research areas that can further current understandings in the area of rational numbers.

RECOMMENDED RESOURCES

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